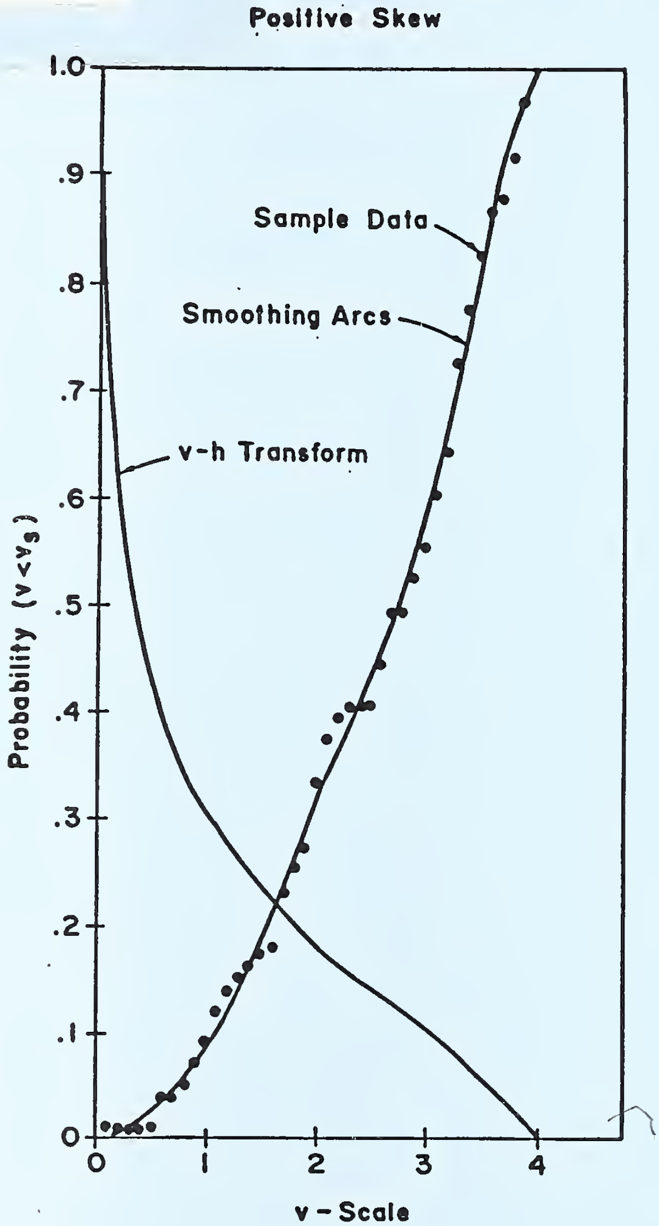
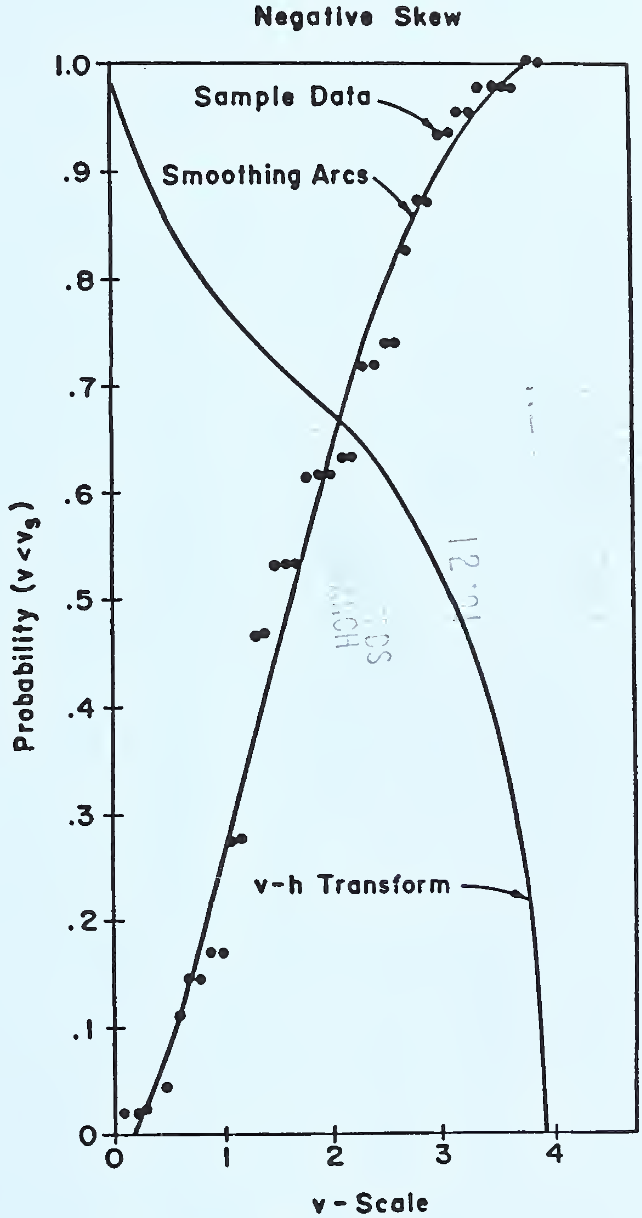
A COMPUTER PROGRAM FOR TRANSFORMING STOCHASTIC DATA AND EVALUATING PROBABILITY DISTRIBUTIONS





Southern Piedmont Conservation Research Center
Agricultural Research Service, USDA
Watkinsville, Georgia 30677

RESEARCH REPORT
No. IRC 010186
January 1986

Research Report[1/]
No. IRC 010186

A COMPUTER PROGRAM FOR TRANSFORMING STOCHASTIC DATA AND EVALUATING PROBABILITY DISTRIBUTIONS

A. W. Thomas
Agricultural Engineer
Southern Piedmont Conservation Research Center
Agricultural Research Service, USDA
Watkinsville, GA 30677

W. M. Snyder
Hydrologic Consultant
275 Gatewood Circle
Athens, GA 30607

and

A. L. Dillard
Agricultural Engineer (Graduate Co-Op Student)
Southern Piedmont Conservation Research Center
Agricultural Research Service, USDA
Watkinsville, GA 30677

January 1986

[1/] This report describes a computer program and its operational details which transforms stochastic data and evaluates probability distributions. This methodology was used in recent papers by Snyder and Thomas, 1986; and Thomas and Snyder, 1986a, 1986b.

INTRODUCTION

A data transformation method was developed and tested which appears to be useful with a wide variety of stochastic data over a wide range of sample characteristics. The transform is essentially exponential and allows reflection of one-side infinite data into a finite domain with proper boundary conditions. The mathematical transform function is scaled and shaped by the sample mean, standard deviation, and coefficient of skew. The method provides a means for visual display of data with slight curvilinearity for simple samples, or for more complex samples exhibiting such characteristics as bi-modal tendency, positive or negative skew, and presence or absence of outliers.

In an earlier research report, Thomas and Snyder (1984c) reported a mathematically form-free method for performing probablistic analysis and synthesis of experimental data without assuming any particular conventional distribution function. They had successfully used the methodology in studies reported in three papers (Snyder and Thomas, 1983; Thomas and Snyder, 1984a, 1984b). Additional background material on sliding polynomials which is used in the related smoothing operation may be found in papers by Snyder (1976 and 1980).

We stated in the earlier report (Thomas and Snyder, 1984c) that the transform would be gradually standardized through additional research and experience. The need for improvement of the transform became very clear as we tried to develop seasonally continuous probability distributions (Snyder and Thomas, 1986; and Thomas and Snyder, 1986b). The old version of the transform proved inadequate as we tried to bring into alignment the samples from different months of

the year to provide nearly homogeneous data that could be smoothed by a mathematical surface. The experience and success of the improved transform are reported in the following three papers: Snyder and Thomas, 1986; and Thomas and Snyder, 1986a, 1986b.

The description and computer program of the improved transform are provided in this report for those that are interested.

TRANSFORM STRUCTURE

The mathematical transform consists of two steps. First, the historical variate, h, is reduced to a standardized variate, h'. Secondly, h' is converted to an abstract variate, v, using an ogee-shaped curve composed of two limbs of the common descending exponential, joined at a common point, C, as shown in Fig. 1. These two limbs are both shifted and shaped with the skewness of the sample. The addition of the skewness parameter improved the transform and contributed significantly to its enhanced capabilities. Changes in the transform for positive skew are shown in Fig. 1 while changes with negative skew are idential but reversed.

Positive Skew. The v_1 and v_2 limbs of the transform are given in equation [1].

$$\begin{aligned} v_1 &= VB - (VB - C)\exp(f\,h'), & v &\geq C \\ v_2 &= C\exp(-d\,h'), & v &< C \end{aligned} \qquad [1]$$

where C is the common point of limbs in v-scale, VB is the asymptotic right boundary of the v_1 limb, f and d are shape parameters of the limbs, and h' will be discussed later. The linear shift of C with the coefficient of skew, SK, was calibrated by setting C = 2 for zero skew and C = 3.5 for a coefficient of skew of 4. Although this calibration is judgmental, it is based on the geometrical restrictions of the v-scale in Fig. 1 and has, in our experience, performed well. With this calibration, C is defined by equation [2].

$$C = 2 + 0.375\ SK \qquad [2]$$

The boundary of v, VB, shifts the same amount to the right as C and starts from VB = 4.05 for zero skew. We have found this to be a

rational value beyond the required finite value of 4.0 for the calibration point. The expression for VB is given in equation [3].

$$VB = 4.05 + (C - 2.0) \qquad [3]$$

The h' in equation [1] is a re-scaled value of the variate h. The re-scaling reported earlier by Thomas and Snyder, 1984c, utilized only the mean and standard deviation of h. We later found this to be insufficient re-scaling of the variate for highly skewed samples because of insufficient reduction in curvilinearity of the sample when plotted against probability. The solution, we found, was to shift the h and h' scales relative to each other as sample skew changed. This shifting is represented in Fig. 1 and equation [4].

$$h' = \frac{h - \bar{h} + a + b\ SK}{s_h} \qquad [4]$$

where $\bar{h}$ is the mean of the sample under analysis and s_h is its standard deviation. Equation [4] includes important elements of the earlier version but adds a shift controlled by skew. The a and b are calibration parameters and have been judgmentally evaluated by making the shift zero for zero skew and one-half the standard deviation for SK = 4. Equation [5] results and may be considered the operating equation for re-scaling h to h'.

$$h' = \frac{h - \bar{h}}{s_h} + \frac{SK}{8} \qquad [5]$$

The last step is to calibrate the shape parameters, f and d, of the two exponential limbs given in equation [1]. This calibration process requires the selection of an h_{min} value which is one coordinate of a calibrating point for the v_1 limb and is discussed below. The second coordinate for this point is $v_1 = 4$.

Substituting these coordinate values in the v_1 limb of equation [1] yields parameter f as shown in equation [6].

$$f = \ln\left((4 - VB)/(C - VB)\right)/h'_{min} \qquad [6]$$

h'_{min} is computed by setting $h = h_{min}$ in equation [5]. Differentiating equation [1] and setting the derivatives equal at C where $h' = 0$, yields the solution for d given in equation [7].

$$d = f\,(VB - C)/C \qquad [7]$$

The compound curve for positively skewed samples is thus calibrated and its shape is based on sample mean, standard deviation, and coefficient of skew. The only decision by the user is the selection of a value for h_{min}. To select an appropriate value, one should clearly visualize the operating effect and function of h_{min}. The h_{min} value sets the lower boundary of the class containing the smallest events. This corresponds to the requirement of setting the first class boundary when preparing sample histograms in simplistic data reduction. The specification of h_{min} does not establish a lower limit of the variate h.

Negative Skew

The transform for negatively skewed samples is identical but reversed to that represented in Fig. 1 for positive skew. Small values of the historical variate, h, can now extend to negative infinity. The common point, C, moves to the left with increasing negative skew. The limiting boundary, VB, shifts from upper right to lower left in the lower portion of Fig. 1. The calibration point ($v_1 = 4$, h_{min}) for positive skew changes to ($v_2 = 0$, h_{max}) for negative skew where h_{max} is now an input. With these differences, the equivalent of equation [1] becomes equation [8] for negative skew.

$$v_1 = 4 - (4 - C) \exp (f\, h'), \quad v \geq C$$
$$v_2 = (C + VB) \exp (-d\, h') - VB, \; v < C \qquad [8]$$

Equation [2] which shifts the common point, C, does not change except SK is negative. The new equation for VB becomes equation [9].

$$VB = 0.05 + (2.0 - C) \qquad [9]$$

The re-scaling of h to h' is unchanged from that shown in equation [5] except for the negative value of SK. Parameter d of equation [8] is obtained through calibration to give equation [10].

$$d = - \ln (VB/(C + VB))/h'_{max} \qquad [10]$$

h'_{max} in equation [10] is calculated from the input h_{max} using equation [5]. Requiring mathematical continuity at point C gives parameter f as shown in equation [11].

$$f = d\, (C + VB)/(4 - C) \qquad [11]$$

A similar analogy exists for the selection of h_{max} value to that given for the selection of h_{min}.

SMOOTHING

Once the data have been transformed by the above method, any number of smoothing procedures could be readily adapted. However, we oriented the transform development towards a computer-based form-free optimization-smoothing procedure known as sliding polynomials. We smooth the data in the cumulative probability domain, rather than the probability density domain, as demonstrated in Fig. 1. The smoothing procedure in this report is similar to that described by Thomas and Snyder, 1984c, with a few noteworthy exceptions.

The transform allows for reflection of any values of the variate, h, to values of the intermediate variate, v. With this transformation, it is possible to tally a sample of items, h, into

classes which are linear in v-scale as follows. The sample space in v, $0 < v \leq 4$ is divided into 40 equal classes, the class boundaries are reflected to h-scale, and the sample h is tallied into these 40 classes. The class subtotals are accumulated from largest to smallest h. The class accumulations are divided by the sample size, producing accumulated class totals as ratios running from zero to one across four polynomial spans, 0-1, 1-2, 2-3 and 3-4. The earlier method (Thomas and Snyder, 1984c) used only 30 classes with three polynomial spans. The increase in the number of classes and spans improved the ability to smooth complex curvilinearity found in samples reported by Snyder and Thomas, 1986; and Thomas and Snyder, 1986a, 1986b.

Since the accumulation is from large h to small h, the class subtotals of probability must be plotted at the right-hand class boundaries in v-scale. This procedure obviates the long-standing problem of "plotting positions" for individual items of the sample. Not all classes of the 40 specified will necessarily be occupied, particularly for small samples. Consequently, "plateaus" occur in the sample accumulated probabilities which must be smoothed with the arcs of the sliding polynomials. We should note that smoothing results from minimizing errors in the probability dimension which is represented by the ordinate in the top part of Fig. 1. Optimization does not occur in the scale of the transformed variate, v, which is represented by the abscissa.

DISCUSSION

Two historical samples were selected to demonstrate the results of the transformation and smoothing. The first sample was a 99-year record of total rainfall during November at Athens, GA. The second sample was a 47-year record of minimum temperature for November at the Southern Piedmont Conservation Research Center at Watkinsville, GA. Table 1 presents the sample characteristics of mean, standard deviation, skew, and range for the two samples. The transformation parameters of C, VB, f and d are also included. The selected input values of h_{min} and h_{max} for the positive and negative skew, respectively, are also shown.

The rainfall and temperature samples are plotted as accumulated probabilities in Figs. 2 and 3, respectively. Specifically for each figure, 40 class ratios are plotted against the class limit in v-scale. For the positive-skew rainfall sample, Fig. 2, the smoothing line must pass through v = 0, P = 0 with zero slope. In contrast for the negative-skew temperature sample, Fig. 3, the smoothing line must pass through v = 4, P = 1 with zero slope. Super-imposed on the sample plots and derived smoothing lines are the v-h transforms for each sample. Those plotted transforms are derived from equations [1] and [5] for positive skew in Fig. 2 and from equations [8] and [5] for negative skew in Fig. 3. The utility of the v-h transform is exemplified by transformation of the median value. From 0.5 probability, one goes horizontally to the smoothing curve on the sample, then vertically to the v-h transform, then again horizontally to the scale of the original variate h. The mean of h is also shown for comparison.

An APPENDIX is included which gives information and instructions designed to assist a user of this methodology. The section on INPUT VARIABLES provides the code for the input variable name with a brief comment. Likewise, the section on OUTPUT VARIABLES gives the code for the output variable name with a brief comment. Next is the program listing, sectionalized with remark statements. Last, is the sample output for the analysis of the November rainfall which had a positive skew. Not shown is the sample output for the November temperature.

In conclusion, this research report presents a variate transformation that appears to have great potential for use with many types of stochastic distributions. The method is oriented toward the piece-wise method of sliding polynomials; however, it is not limited to this particular methodology. The method has proved useful for simple samples as well as more complex samples exhibiting such characteristics as bi-modal tendency, positive or negative skew, and presence or absence of outliers.

REFERENCES

1. Snyder, Willard M. 1976. Interpolation and Smoothing of Experimental Data with Sliding Polynomial. USDA, Agricultural Research Service, ARS-S-83. 34 p.

2. Snyder, Willard M. 1980. Smoothed Data and Gradients Using Sliding Polynomials with Optional Controls. Water Resourc. Bull. 16(1):22-30.

3. Snyder, W. M. and A. W. Thomas. 1983. Return Period Analysis with Sliding Polynomials. Trans. ASAE. 26(6):1732-1737.

4. Snyder, W. M. and A. W. Thomas. 1986. Stochastic Impacts on Farming: II. Seasonally Continuous Probability Distributions. Trans. ASAE. In review.

5. Thomas, A. W. and W. M. Snyder. 1984a. Return Period Analysis of Minimum Events Using Sliding Polynomials. Trans. ASAE. 27(2):464-469.

6. Thomas, A. W. and W. M. Snyder. 1984b. Testing the Representativeness of Short Period Records through Simulation. Trans. ASAE. 27(4):1027-1033.

7. Thomas, A. W. and W. M. Snyder. 1984c. Computer Programs for Analysis and Simulation of Probability Distributions Using Sliding Polynomials. Technical Research Report No. IRC 070184. Watkinsville, GA.

8. Thomas, A. W. and W. M. Snyder. 1986a. Stochastic Impacts on Farming: I. Transformation of Relevant Variates. Trans. ASAE. Accepted for publication.

9. Thomas, A. W. and W. M. Snyder. 1986b. Stochastic Impacts on Farming: III. Simulation of Seasonal Variation of Climatic Risk. Trans. ASAE. In review.

TABLE 1. CHARACTERISTICS OF TEST DATA

	November Total Monthly Rainfall		November Minimum Temperature	
Mean*	3.95	(7.77)	23.87	(4.52)
Std. Dev.*	2.33	(5.92)	5.08	(2.82)
Coef. of Skew	2.06		-0.99	
Range*	15.78- 0.33	(40.08- 0.84)	35- 5	(1.67- -15.00)
C**	2.77		1.63	
VB**	4.82		0.42	
f**	0.86		0.61	
d**	0.64		0.70	
h_{min}*	0	(0)	---	
h_{max}*	---		36.00	(2.22)

*Units: rainfall, inch (cm); temperature, °F (°C).

**Definitions: C - common point, VB - asymptotic boundary, f and d - shape parameters.

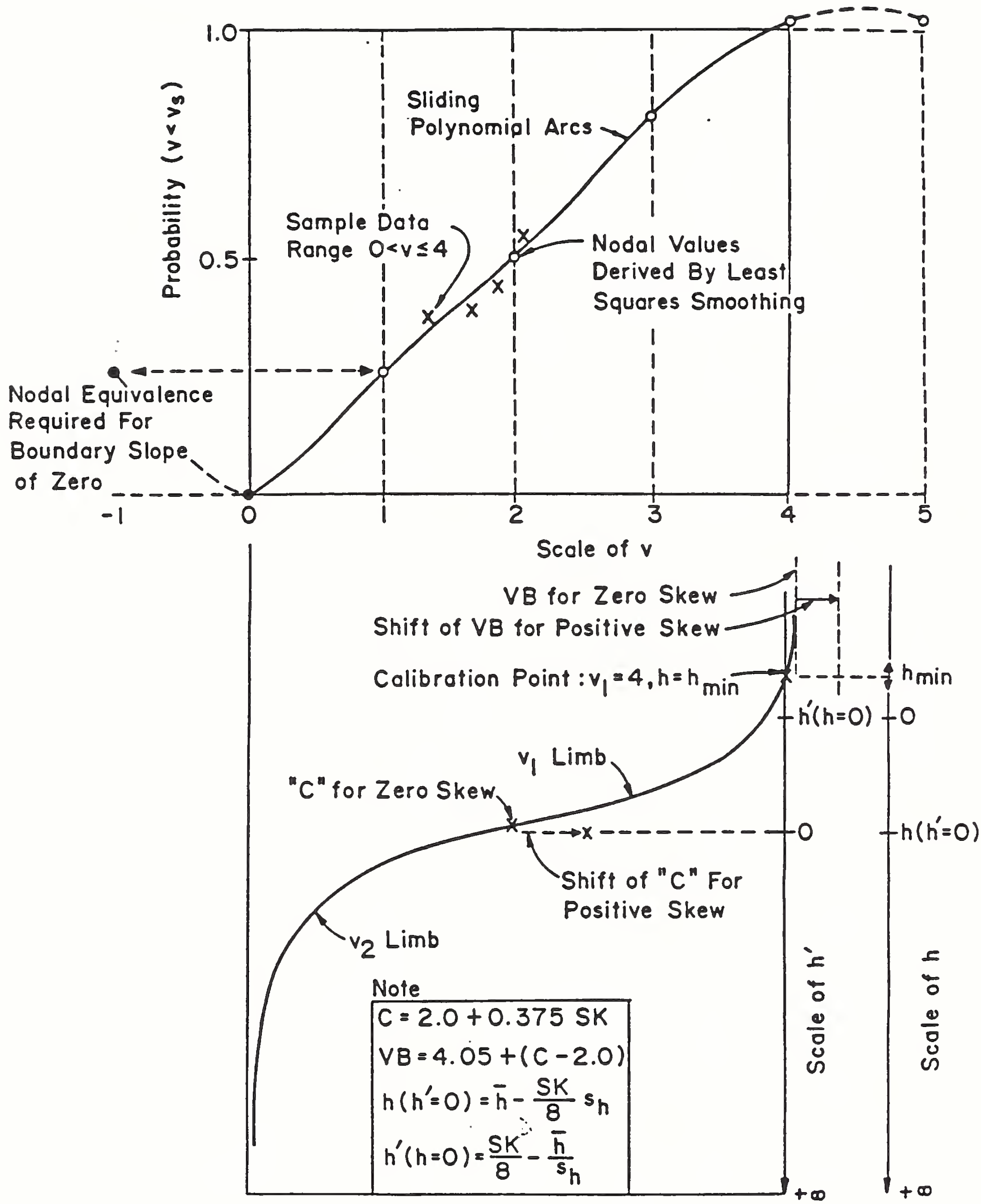


Fig. 1 Schematic for variate transformation and smoothing.

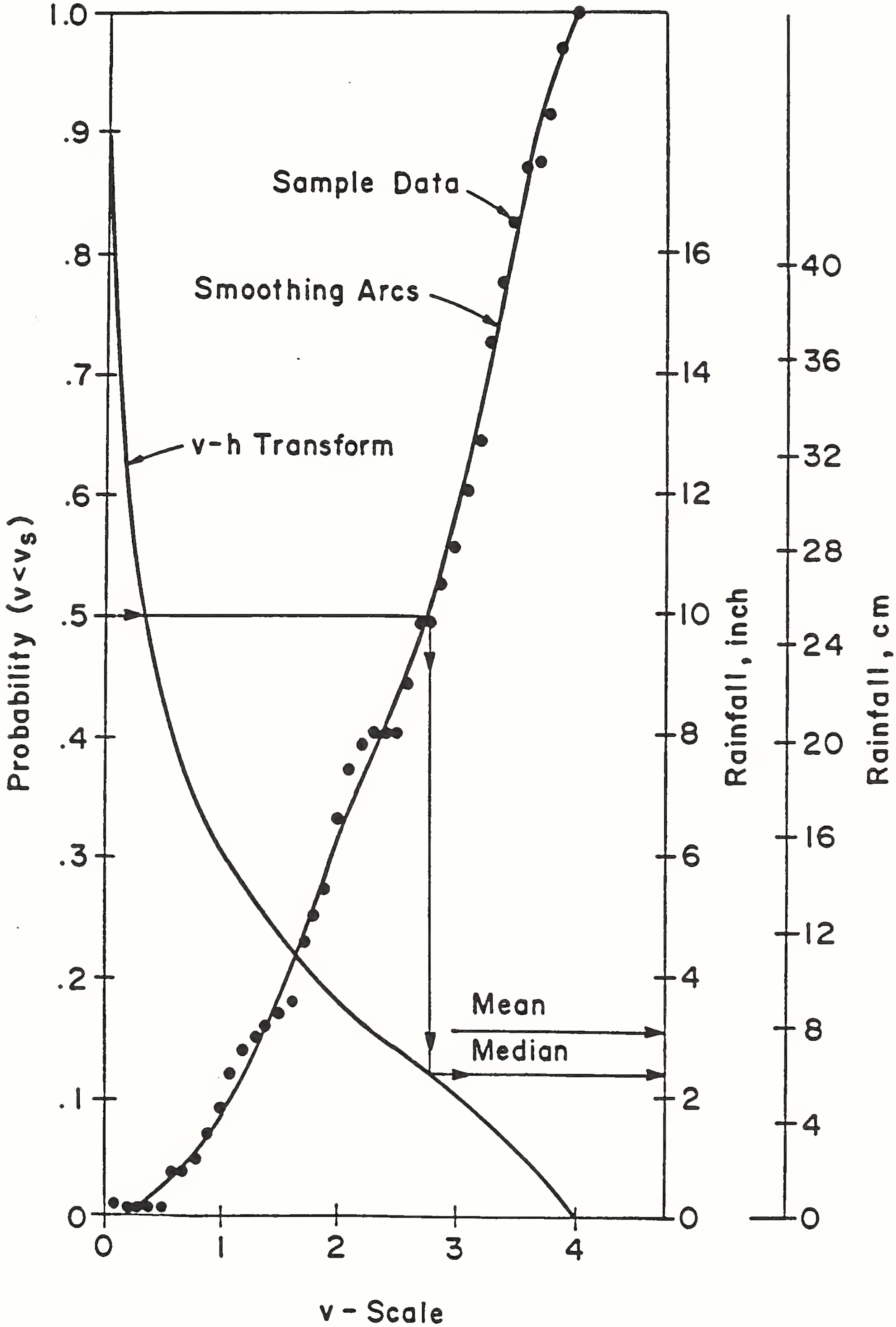


Fig. 2 Transformed and smoothed sample of total monthly rainfall for November at Athens, GA.

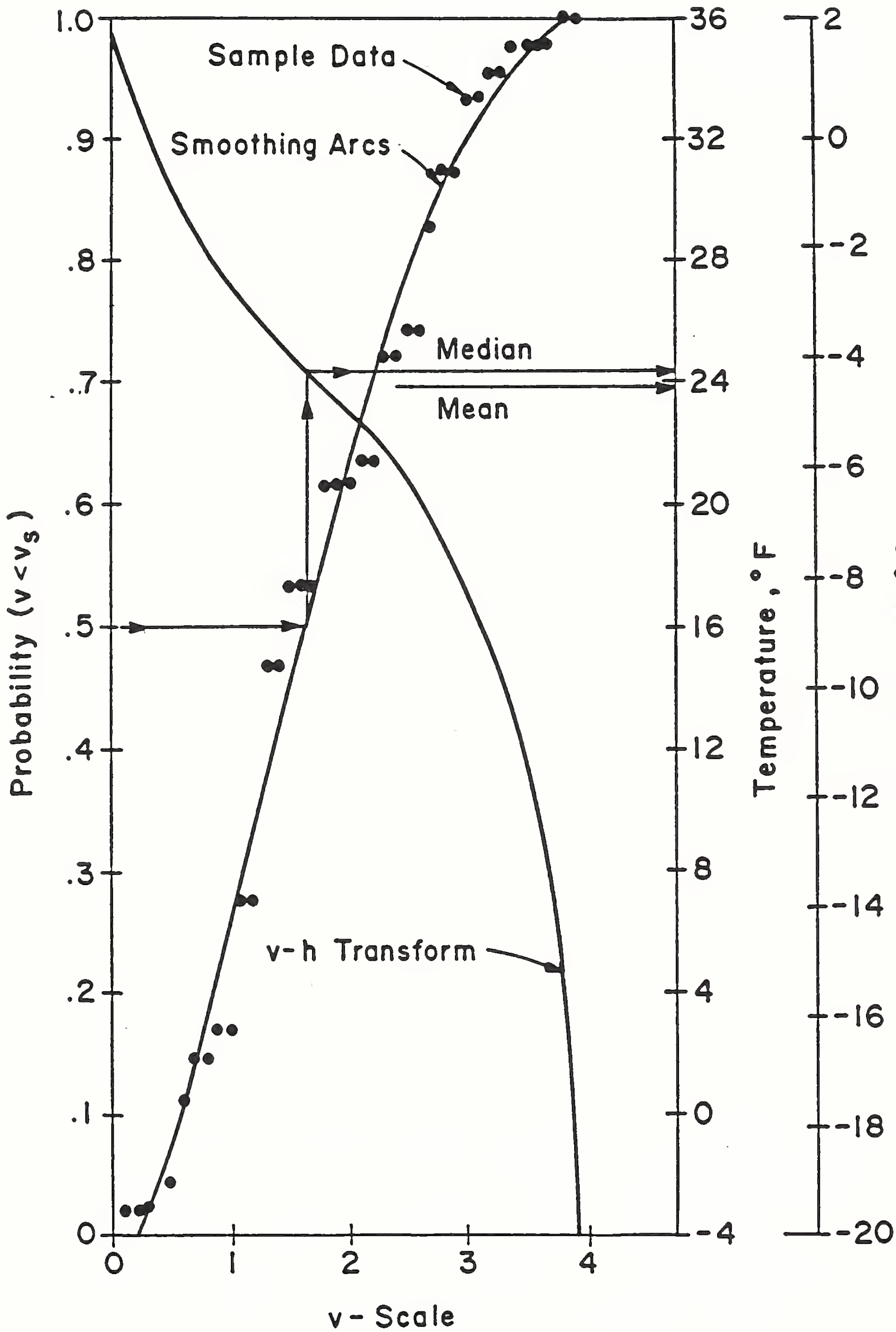


Fig. 3 Transformed and smoothed sample of minimum monthly temperature for Nov at Watkinsville, GA.

APPENDIX

The Appendix includes a description of input and output variables, program listing, and sample output of the analysis of the November total rainfall.[2]

---

[2]The program is presented for the convenience of potential users. While the program has been run and tested on various data sets, the originators of the program assume no responsibility for its accuracy or adequacy. Such responsibility must rest solely on the user. We stand ready to assist and advise within the limitations imposed by our operating resources. The program is listed in Hewlett-Packard BASIC with Ext. 2.1. (Trade name is included for the benefit of the reader and does not imply an endorsement or preferential treatment of the named product.)

Input Variables

Variable name	Comment
PRTR	Printer output device control. (1=CRT, 706=Printer)
T$	Problem title
HM	Minimum class limit in data scale
HH	Maximum class limit in data scale
DIC	Data input control (1=Keyboard input, 2=Disk resident)
N	Number of data points
DSN$	Disk resident data file name
H(I); I=1,N	Data points

Output Variables

Variable Name	Comments
T$	Problem title
HM	Minimum class limit in data scale
HH	Maximum class limit in data scale
HB	Sample mean
SSD	Sample standard deviation
SK	Sample coefficient of skew
C1	Common point of exponential limbs
VB	Asymptotic boundary
F	Shape parameter
D	Shape parameter
CH(I):I=1,40	Class limit in h-scale
CT(I):I=1,40	Sample class probabilities
SC(I,6):I=1,5	'Best Fit' nodal ordinates
CP(I):I=1,40	Smoothed class probabilities
SD(I):I=1,5	Standard deviation of nodes

Program Listing

```
10   REM---------1D PROBABILITY ANALYSIS----------------------
20   REM---------4 SPANS WITH NEW TRANSFORM-------------------
30   REM---------HANDLES POS AND NEG SKEW---------------------
40   OPTION BASE 1
50   DIM H(100),CH(40),CT(40),C(40,7),SC(6,6),CP(40),E(40),SD(5)
60   DIM T$[50],DSN$[30]
70   INPUT "TO HARDCOPY PRINT ENTER(706), CRT ENTER(1)",PRTR
80   LINPUT " PROBLEM TITLE",T$
90   PRINT "MIN H USED WITH 0 OR + SKEW, MAX H USED WITH - SKEW"
100  INPUT "MINIMUM VALUE CLASS LIMIT OF H",HM
110  INPUT "MAXIMUN VALUE CLASS LIMIT OF H",HH
120  REM----INPUT DATA CONTROL IS '1' FOR KEY-IN '2' FOR DISK---
130  INPUT "DATA CONTROL: ENTER(1) FOR KEY-IN, (2) FOR DISK",DIC
140  OUTPUT PRTR;"       NEW TRANSFORM WITH FOUR ARCS"
150  OUTPUT PRTR;T$
160  OUTPUT PRTR;"MINIMUM VALUE CLASS LIMIT OF H  ";HM
170  OUTPUT PRTR;"MAXIMUN VALUE CLASS LIMIT OF H  ";HH
180  IF DIC=2 THEN 240
190  INPUT "NUMBER OF DATA POINTS",N
200  FOR I=1 TO N
210  INPUT H(I)
220  NEXT I
230  GOTO 310
240  LINPUT " DATA SET NAME ",DSN$
250  ASSIGN @INP TO DSN$
260  ENTER @INP;N
```

```
270   FOR I=1 TO N
280   ENTER @INP;H(I)
290   NEXT I
300   ASSIGN @INP TO *
310   REM---------------FIND FIRST THREE MOMENTS----------------
320   SH=0.
330   SHH=0.
340   SHHH=0.
350   FOR I=1 TO N
360   SH=SH+H(I)
370   XH=H(I)*H(I)
380   SHH=SHH+XH
390   SHHH=SHHH+XH*H(I)
400   NEXT I
410   HB=SH/N
420   SSD=SQR(SHH/N-HB*HB)
430   SK=((SHHH/N)-(3*HB*SHH/N)+(2*HB*HB*HB))/SSD/SSD/SSD
440   OUTPUT PRTR;" MEAN ";HB;"  STD DEV  ";SSD;"  COEF OF SKEW  ";SK
450   REM---------TEST FOR NEGATIVE OR POSITIVE SKEW------------
460   IF SK<0. THEN 810
470   REM------------------POSITIVE SKEW-----------------------
480   HPM=(HM-HB)/SSD+SK/8
490   C1=2.+.375*SK
500   VB=4.05+(C1-2.)
510   F=LOG((4-VB)/(C1-VB))/HPM
520   D=F*(VB-C1)/C1
530   V=0.
```

```
540   OUTPUT PRTR;"C";C1;"   VB";VB;"   F";F;"   D";D
550   REM----------------CALCULATE CLASS LIMITS-----------------
560   FOR I=1 TO 39
570   V=V+.1
580   IF V>C1 THEN 610
590   CH(I)=-SSD*(LOG(V/C1)/D+SK/8)+HB
600   GOTO 620
610   CH(I)=SSD*(LOG((V-VB)/(C1-VB))/F-SK/8)+HB
620   NEXT I
630   CH(40)=HM
640   REM----------------CALCULATE SAMPLE PROBABILITIES---------
650   FOR J=1 TO N
660   FOR I=1 TO 40
670   IF H(J)>=CH(I) THEN 690
680   GOTO 710
690   CT(I)=CT(I)+1
700   GOTO 720
710   NEXT I
720   NEXT J
730   FOR J=2 TO 40
740   CT(J)=CT(J)+CT(J-1)
750   NEXT J
760   FOR I=1 TO 40
770   CT(I)=CT(I)/N
780   NEXT I
790   GOTO 1140
800   REM------------------NEGATIVE SKEW-----------------------
```

```
810   HPM=(HH-HB)/SSD+SK/8
820   C1=2.+.375*SK
830   VB=.05+(2.-C1)
840   D=-LOG(VB/(C1+VB))/HPM
850   F=D*(C1+VB)/(4-C1)
860   OUTPUT PRTR;"C";C1;"   VB";VB;"   F";F;"   D";D
870   REM------------CALCULATE CLASS LIMITS---------------------
880   V=0.
890   FOR I=2 TO 40
900   V=V+.1
910   IF V>C1 THEN 940
920   CH(I)=SSD*(-LOG((V+VB)/(C1+VB))/D-SK/8)+HB
930   GOTO 950
940   CH(I)=SSD*(LOG((4-V)/(4-C1))/F-SK/8)+HB
950   NEXT I
960   CH(1)=HH
970   REM------------CALCULATE SAMPLE PROBABILITIES--------------
980   FOR J=1 TO N
990   FOR I=2 TO 40
1000  IF H(J)>=CH(I) THEN 1020
1010  GOTO 1040
1020  CT(I)=CT(I)+1
1030  GOTO 1050
1040  NEXT I
1050  NEXT J
1060  FOR J=2 TO 40
1070  CT(J)=CT(J)+CT(J-1)
```

```
1080  NEXT J
1090  FOR I=2 TO 40
1100  CT(I)=CT(I)/N
1110  NEXT I
1120  CT(1)=0.
1130  REM-------------------------------------------------------
1140  OUTPUT PRTR;"CLASS LIMITS AND SAMPLE CLASS PROBABILITIES"
1150  J=1
1160  FOR I=1 TO 40
1170  IMAGE #,3D,7D.DD,2D.3D
1180  OUTPUT PRTR USING 1170;I,CH(I),CT(I)
1190  IF J=3 THEN 1230
1200  J=J+1
1210  OUTPUT PRTR;"    ";
1220  GOTO 1250
1230  J=1
1240  OUTPUT PRTR
1250  NEXT I
1260  OUTPUT PRTR
1270  REM-------------- LOCATE SEVEN NODES-----------------------
1280  XN(1)=1
1290  FOR I=2 TO 7
1300  XN(I)=XN(I-1)+1
1310  NEXT I
1320  REM-------------CALCULATE COEFFICIENT C(I,J)---------------
1330  REM----------------7 FOR EACH OF 40 CLASSES----------------
1340  FOR IP=1 TO 40
```

```
1350  JC=1
1360  IF (IP*.1)<=JC THEN 1390
1370  JC=JC+1
1380  GOTO 1360
1390  DP=(IP*.1)-JC+1.
1400  Z=-.5+DP
1410  IF SK<0. THEN Z=Z-.1
1420  C(IP,JC)=(((-8*Z+4)*Z+2)*Z-1)/16
1430  C(IP,JC+1)=(((24*Z-4)*Z-22)*Z+9)/16
1440  C(IP,JC+2)=(((-24*Z-4)*Z+22)*Z+9)/16
1450  C(IP,JC+3)=(((8*Z+4)*Z-2)*Z-1)/16
1460  NEXT IP
1470  IF SK<0. THEN 1620
1480  REM------IF POS. SKEW, PUT IN LEFT BOUNDARY ZERO AND------
1490  REM------ZERO SLOPE, RIGHT BOUNDARY IS FREE--------------
1500  FOR IP=1 TO 40
1510  C(IP,3)=C(IP,3)+C(IP,1)
1520  NEXT IP
1530  REM------MOVE TO LEFT AND ADD CLASS PROB TO MATRIX---------
1540  FOR IP=1 TO 40
1550  FOR J=1 TO 5
1560  C(IP,J)=C(IP,J+2)
1570  NEXT J
1580  C(IP,6)=CT(IP)
1590  NEXT IP
1600  GOTO 1720
1610  REM------IF NEG. SKEW, PUT IN RIGHT BOUNDARY 1.0 AND------
```

```
1620 REM------ZERO SLOPE, LEFT BOUNDARY IS FREE---------------
1630 FOR IP=1 TO 40
1640 C(IP,5)=C(IP,5)+C(IP,7)
1650 NEXT IP
1660 REM------ADD CLASS PROBABILITIES AND NODE 6=1.0------------
1670 REM-------------BOUNDARY; SAVE C(IP,6) IN 7-------------
1680 FOR IP=1 TO 40
1690 C(IP,7)=C(IP,6)
1700 C(IP,6)=CT(IP)-C(IP,7)
1710 NEXT IP
1720 REM---------CALCULATE SUMS OF SQUARES MATRIX-------------
1730 FOR K=1 TO 40
1740 FOR I=1 TO 6
1750 FOR J=I TO 6
1760 SC(I,J)=SC(I,J)+C(K,I)*C(K,J)
1770 NEXT J
1780 NEXT I
1790 NEXT K
1800 PRINT "SUMS OF SQUARES CALCULATED"
1810 REM-------------FILL IN COMPLETE MATRIX-----------------
1820 FOR I=1 TO 5
1830 FOR J=1 TO 5
1840 SC(J,I)=SC(I,J)
1850 NEXT J
1860 NEXT I
1870 REM-------------SOLVE SET OT 5 EQUATIONS----------------
1880 FOR J=1 TO 5
```

```
1890 FOR I=J TO 5
1900 IF SC(I,J)<>0. THEN 1940
1910 NEXT I
1920 PRINT "NO UNIQUE SOLUTION"
1930 GOTO 2180
1940 FOR K=1 TO 6
1950 X=SC(J,K)
1960 SC(J,K)=SC(I,K)
1970 SC(I,K)=X
1980 NEXT K
1990 Y=1/SC(J,J)
2000 FOR K=1 TO 6
2010 SC(J,K)=Y*SC(J,K)
2020 NEXT K
2030 FOR I=1 TO 5
2040 IF I=J THEN 2090
2050 Y=-SC(I,J)
2060 FOR K=1 TO 6
2070 SC(I,K)=SC(I,K)+Y*SC(J,K)
2080 NEXT K
2090 NEXT I
2100 NEXT J
2110 OUTPUT PRTR;"  SOLUTION FOR FIVE BEST FIT NODES"
2120 FOR I=1 TO 5
2130 OUTPUT PRTR USING "3D,K,DD.5D,K";I,"    ",SC(I,6),"   "
2140 NEXT I
2150 OUTPUT PRTR
```

```
2160 OUTPUT PRTR
2170 REM-------LAY IN SLIDING POLYNOMIAL THROUGH NODES---------
2180 OUTPUT PRTR;"SMOOTHED CLASS TOTAL PROBABILITIES"
2190 IF SK<0. THEN 2380
2200 FOR I=1 TO 40
2210 IF I>30 THEN 2330
2220 IF I>20 THEN 20
2230 IF I>10 THEN 2260
2240 CP(I)=C(I,1)*SC(1,6)+C(I,2)*SC(2,6)
2250 GOTO 2360
2260 CP(I)=C(I,1)*SC(1,6)+C(I,2)*SC(2,6)+C(I,3)*SC(3,6)
2270 GOTO 2360
2280 CP(I)=0.
2290 FOR J=1 TO 4
2300 CP(I)=CP(I)+C(I,J)*SC(J,6)
2310 NEXT J
2320 GOTO 2360
2330 FOR J=2 TO 5
2340 CP(I)=CP(I)+C(I,J)*SC(J,6)
2350 NEXT J
2360 NEXT I
2370 GOTO 2500
2380 FOR I=1 TO 40
2390 IF I>30 THEN 2480
2400 IF I>20 THEN 2460
2410 IF I>10 THEN 2440
2420 CP(I)=C(I,1)*SC(1,6)+C(I,2)*SC(2,6)+C(I,3)*SC(3,6)+C(I,4)*SC(4,6)
```

```
2430  GOTO 2490
2440  CP(I)=C(I,2)*SC(2,6)+C(I,3)*SC(3,6)+C(I,4)*SC(4,6)+C(I,5)*SC(5,6)
2450  GOTO 2490
2460  CP(I)=C(I,3)*SC(3,6)+C(I,4)*SC(4,6)+C(I,5)*SC(5,6)+C(I,7)
2470  GOTO 2490
2480  CP(I)=C(I,4)*SC(4,6)+C(I,5)*SC(5,6)+C(I,7)
2490  NEXT I
2500  J=1
2510  FOR I=1 TO 40
2520  OUTPUT PRTR USING "#,3D,3D.3D,2X";I,CP(I)
2530  IF J=5 THEN 2570
2540  OUTPUT PRTR;"  ";
2550  J=J+1
2560  GOTO 2590
2570  J=1
2580  OUTPUT PRTR
2590  NEXT I
2600  FOR I=1 TO 40
2610  E(I)=CP(I)-CT(I)
2620  NEXT I
2630  FOR J=1 TO 5
2640  Z2S=0.
2650  Z1S=0.
2660  Z0S=0.
2670  FOR I=1 TO 40
2680  Z2S=Z2S+E(I)*E(I)*C(I,J)*C(I,J)
2690  Z1S=Z1S+C(I,J)*C(I,J)
```

```
2700  Z0S=Z0S+C(I,J)
2710  NEXT I
2720  IF Z0S<=0. THEN 2740
2730  GOTO 2760
2740  SD(J)=99.99
2750  GOTO 2770
2760  SD(J)=SQR(Z2S/Z1S/Z0S*N/(N-5))
2770  NEXT J
2780  OUTPUT PRTR;"  STANDARD DEVIATION OF NODES"
2790  FOR J=1 TO 5
2800  OUTPUT PRTR USING "3D,3D.4D";J,SD(J)
2810  NEXT J
2820  OUTPUT PRTR
2830  OUTPUT PRTR;"       PROBLEM END    "
2840  STOP
2850  END
```

Program Output

```
     NEW TRANSFORM WITH FOUR ARCS
       NOV TOTAL RAIN
MINIMUM VALUE CLASS LIMIT OF H   0
MAXIMUN VALUE CLASS LIMIT OF H   0
 MEAN  3.06353535354  STD DEV   2.32778010743  COEF OF SKEW   2.0566512138
C 2.77124420517   VB 4.82124420517   F .863814250258   D .63899789478
CLASS LIMITS AND SAMPLE CLASS PROBABILITIES
  1      14.57  .010     2     12.04  .010     3     10.56  .010
  4       9.52  .010     5      8.70  .010     6      8.04  .040
  7       7.48  .040     8      6.99  .051     9      6.56  .071
 10       6.18  .091    11      5.83  .121    12      5.51  .141
 13       5.22  .152    14      4.95  .162    15      4.70  .172
 16       4.47  .182    17      4.25  .232    18      4.04  .253
 19       3.84  .273    20      3.65  .333    21      3.48  .374
 22       3.31  .394    23      3.14  .404    24      2.99  .404
 25       2.84  .404    26      2.70  .444    27      2.56  .495
 28       2.43  .495    29      2.29  .525    30      2.15  .556
 31       1.99  .606    32      1.83  .646    33      1.66  .727
 34       1.48  .778    35      1.28  .828    36      1.07  .869
 37        .84  .879    38       .59  .919    39       .31  .970
 40       0.00 1.000
  SOLUTION FOR FIVE BEST FIT NODES
  1      .08569
  2      .31666
  3      .56916
  4      .98832
  5      .75764

SMOOTHED CLASS PROBABILITIES
  1    .001        2    .004        3    .009        4    .015        5    .023
  6    .033        7    .044        8    .057        9    .070       10    .086
 11    .103       12    .122       13    .144       14    .167       15    .191
 16    .216       17    .241       18    .267       19    .292       20    .317
 21    .340       22    .363       23    .386       24    .408       25    .431
 26    .455       27    .480       28    .508       29    .537       30    .569
 31    .607       32    .653       33    .703       34    .756       35    .809
 36    .859       37    .905       38    .943       39    .972       40    .988
  STANDARD DEVIATION OF NODES
  1    .0040
  2    .0071
  3    .0048
  4    .0076
  5 99.9900*
       PROBLEM END
```

*Default for undefined value.